TUTORIAL 8

8.1 FUNCTIONS OF SEVERAL VARIABLES

QUESTION 1

- Let $f(x, y) = x^2 e^{3xy}$. (a) Evaluate f(2, 0).
- Let $f(x, y, z) = e^{\sqrt{z-x^2-y^2}}$. (b) Evaluate f(2, -1, 6).
- Let $g(x, y, z) = \ln(25 x^2 y^2 z^2)$. (c) Evaluate q(2, -2, 4).
- (d) Let $f(x, y) = \ln(x + y - 1)$.
 - (a) Evaluate f(1, 1).
 - (b) Evaluate f(e, 1).

8.2 PARTIAL DERIVATIVES

QUESTION 2

Find the first partial derivatives of the function.

$$f(x, y) = 3x - 2y^4$$

$$f(x, y) = x^5 + 3x^3y^2 + 3xy^4$$

$$z = xe^{3y}$$

$$f(x, y) = \frac{x - y}{x + y}$$

$$w = \sin \alpha \cos \beta$$

$$f(r,s) = r \ln(r^2 + s^2)$$

$$f(x, y) = xe^{xy} - e^{x} \ln(x + y)$$

$$g(x, y) = e^{-x} \sin(x + y)$$

QUESTION 3

Find the indicated partial derivatives.

$$f(x, y) = \sqrt{x^2 + y^2}; \quad f_x(3, 4)$$

$$f(x, y) = \sin(2x + 3y); \quad f_y(-6, 4)$$

8.3 CHAIN RULE

QUESTION 4

Use the chain rule to find $\frac{dz}{dt}$.

(i)
$$z = x^2y + xy^2$$
, $x = 2 + t^4$, $y = 1 - t^3$

(ii)
$$z = \sqrt{x^2 + y^2}$$
, $x = e^{2t}$, $y = e^{-2t}$

(iii)
$$z = \sin x \cos y$$
, $x = \pi t$, $y = \sqrt{t}$

(iv)
$$z = x \ln(x + 2y)$$
, $x = \sin t$, $y = \cos t$

QUESTION 5

Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

(i)
$$z = x^2 + xy + y^2$$
, $x = s + t$, $y = st$

(ii)
$$z = x/y$$
, $x = se^t$, $y = 1 + se^{-t}$